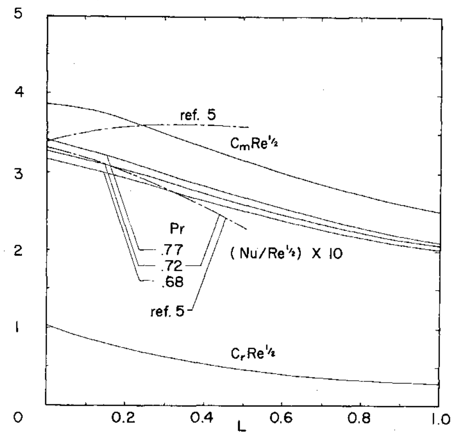

Fig. 3 Drag, friction moment, and heat-transfer coefficients for various conditions of slip.

with the Karman-Pohlhausen method. Cochran[8] demonstrated that, for a continuum situation, the magnitude of the tangential moment-coefficient increased from 3.37 to 3.87 when this restriction is relaxed, which further substantiates the validity of this work (see Fig. 3).

In contrast to the results of the first-order perturbation-analysis of Shidlovskiy as well as the predictions of continuum theory, the results of this study indicate that all of the transport characteristics decrease significantly with increasing rarefaction as should be expected since increasing rarefaction decreases the frequency of molecular interaction at solid boundaries.

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Parachute Critical Opening Velocity

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Nomenclature

a_r = radial (outward) acceleration of canopy element
D_0 = parachute constructed diameter
f_i = functional relationship (i = 1,2,3)
g = acceleration of gravity
l_s = suspension line length
N = number of suspension lines
v_c = parachute critical opening velocity
γ_{avg} = canopy material average mass per unit area
ρ = atmosphere mass density

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Introduction

Parachute critical opening velocity v_c, also called the squidding velocity, is discussed by O'Hara[1] and Brown.[2] Experimental data presented by Brown[2] show that v_c is a function of several geometrical factors (N, l_s/D_0, porosity), as well as of parachute size. In this Note, a rudimentary analysis is used to obtain dimensionless products associated with v_c scale factor (i.e., size) effects.

Analysis

Squidding is essentially a stall or hold in the normal parachute inflation process which occurs at or near the end of the initial phase of inflation.[3,4] The squidded parachute canopy, shown schematically in Fig. 1, is of small cross section and approximately cylindrical. If one considers only geometrically similar parachutes in incompressible flow, the forces per unit area tending to expand and/or collapse the canopy radially are a function of the pressure difference across the canopy, which is in turn a function of $\frac{1}{2}\rho v$.[2] The effective canopy mass per unit area is a function of γ_{avg}. Thus, the outward acceleration of a unit area of canopy is, in dimensionless form,

Table 1 Summary of v_c data

Source	Parachute characteristics[a]: Geometry and mass	D_0, ft	Squid conditions: v_c, fps	ρ, slug/ft³
Brown[2]	porosity = 43 ft³/ft²-sec at 10 in. water	3	95	
	l_s/D_0 = 0.67	$4\frac{1}{2}$	87	
	N = 12	6	58	0.00238[b]
	$g\gamma_{avg}$ = 1.6 oz/yd²[b]	9	65	
		12	50	
Berndt[3]	porosity = 100 ft³/ft²-min at ½ in. water	28	331	0.00159
	l_s/D_0 = 1.0	28	351	0.00159
	N = 28	28	356	0.00159
	$g\gamma_{avg}$ = 1.1 oz/yd²	28	334	0.00124

[a] All chutes are solid flat circular type.
[b] Assumed value.

$$a_r/g = f_1(\tfrac{1}{2}\rho v^2)/gf_2(\gamma_{avg}) \quad (1)$$

The squidding parachute represents an equilibrium condition with $a_r = 0$ and $v = v_c$. These conditions and Eq. (1) suggest the relationship

$$v_c^2/gD_0 = f_3(\gamma_{avg}/\rho D_0) \quad (2)$$

Discussion

The only experimental data which appear available to check the validity of Eq. (2) are summarized in Table 1.

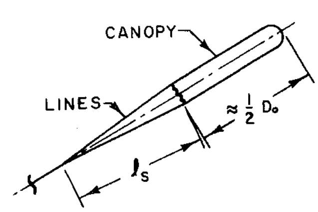

Fig. 1 Squidding parachute.

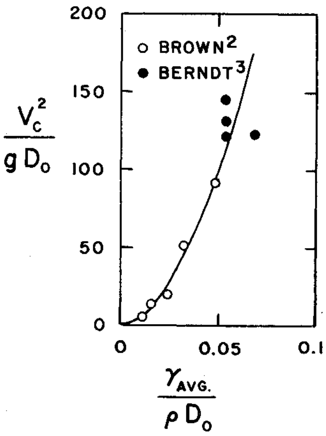

Fig. 2 v_c^2/gD_0 vs $\gamma_{avg}/\rho D_0$.

Figure 2 shows a plot of v_c^2/gD_0 vs $\gamma_{avg}/\rho D_0$ based on the data of Table 1. The equation of the curve in Fig. 2 is

$$v_c^2/gD_0 = (2.74 \times 10^4)(\gamma_{avg}/\rho D_0)^{1.87} \tag{3}$$

Equation (3) indicates that v_c^2 varies approximately directly with $g\gamma_{avg}^2/\rho^2 D_0$, which agrees with Brown's observation[2] that v_c varies approximately as $D_0^{-1/2}$.

As seen, the data of Fig. 2 are reasonably well correlated. However, whether the correlation is real or merely fortuitous remains open to question, since the two sets of data in Table 1 do not meet the constraint of geometrical similarity. This Note is therefore presented in the hope of stimulating further research and experimentation on parachute critical opening velocity.

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Coriolis Coupled Bending Vibrations of Hingeless Helicopter Rotor Blades

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Nomenclature

$(EI)_v$ = in-plane flexural rigidity, lb-ft^2
$(EI)_w$ = out-of-plane flexural rigidity, lb-ft^2
L = lift per unit length of blade, lb/ft
Q = axial tension force, lb
R = radius of rotor, span of rotor blade, ft
V = in-plane blade displacement, ft
W = out-of-plane blade displacement, ft
a_i, b_i = coefficients in Galerkin series expansion
a_v, a_w = absolute acceleration components in-plane and out-of-plane, respectively, ft/sec^2
m = mass per unit length, slug/ft
r = radial position along blade, ft
t = time, sec
v = amplitude of in-plane displacement, ft
w = amplitude of out-of-plane displacement
x = dimensionless blade span coordinate
β = preconing angle of blade with respect to plane of rotation, rad
γ = coupling factor in characteristic equation
ξ = dummy variable of integration, ft
η = dimensionless dummy variable of integration
ψ = blade azimuth coordinate, rad
ν = natural frequency ratio, cycles/revolution
Ω = frequency of steady rotation of blade, rad/sec
$'$ = differentiation with respect to span coordinate or dimensionless span coordinate
$\cdot$ = differentiation with respect to time

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Introduction

The bending vibrations of conventional articulated helicopter rotor blades are usually considered to be uncoupled oscillations, involving small displacements normal to or in the plane of steady rotation. The resulting modes of vibration are similar to those obtained for a nonuniform, pin-free beam, modified by the additional inertia loading of the centrifugal force field. In the case of the hingeless or cantilever rotor blade, it is necessary to utilize an initial "preconing" or built-in root slope to minimize the steady bending moment caused by the lift loading of the blade by counteracting it with one caused by the centrifugal force. This preconing results in a Coriolis inertia loading which couples the out-of-plane and in-plane bending vibrations, leading in some cases of practical interest to significant differences in the coupled natural frequencies of vibration relative to those predicted by the usual uncoupled vibration analysis.

Analysis

For the small oscillations of interest, we may utilize the equation of equilibrium of a beam column,[1] modified to account for the varying axial tension along the span caused by centrifugal force and the appropriate transverse inertia and lift loadings. The equations of equilibrium for small blade motions normal to the plane of rotation and in the plane of rotation, respectively, are

$$[(EI)_w W'']'' - (QW')' + ma_w = L(r,t) \tag{1}$$

$$[(EI)_v V'']'' - (QV')' + ma_v = 0 \tag{2}$$

$$Q = \Omega^2 \int_r^R m(\xi)\xi d\xi \tag{3}$$

$$a_w = \ddot{w} + 2\Omega\dot{v}\beta + \Omega^2 r\beta \tag{4}$$

$$a_v = \ddot{v} - \Omega^2 v - 2\Omega\dot{w}\beta \tag{5}$$

Equation (1) corresponds to the more familiar one for the out-of-plane vibration,[2] except for the Coriolis coupling term and the transverse loading caused by the small preconing angle β and centrifugal force. The latter effect is considered by assuming that β has been selected such that

$$\Omega^2 r\beta = L_{steady} \tag{6}$$

This is a good approximation at the normal design loading where the steady lift loading is virtually a linear function of span except at the most outboard blade stations due to the blade trailing tip vortex.

The reduced equations of motion are now solved for the modes of free vibration by first nondimensionalizing and then separating[3] the coupled partial differential equations.

Let $r = Rx$, $\psi = \Omega t$, and

$$W(x,\psi) = w(x)\sin\nu\psi \tag{7}$$

$$V(x,\psi) = v(x)\cos\nu\psi \tag{8}$$

$$L(x,\psi) = L_{steady} = \Omega^2 Rx\beta \tag{9}$$